Coexistence of Regular Undamped Nuclear Dynamics with Intrinsic Chaoticity

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Abstract

We study the conditions under which the nucleons inside a deformed nucleus can undergo chaotic motion. To do this we perform self-consistent calculations in semiclassical approximation utilizing a multipole-multipole interaction of the Bohr-Mottelson type for quadrupole and octupole deformations. For the case of harmonic and non-harmonic static potentials, we find that both multipole deformations lead to regular motion of the collective coordinate, the multipole moment of deformation. However, despite this regular collective motion, we observe chaotic single particle dynamics.

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The question about the origin of dissipation in collective motion of finite Fermi systems [1] such as atomic nuclei or small metallic clusters is an intriguing and up to now not completely satisfactorily solved problem. For example, the mutual balance of one-body and two-body processes is still a question of debate. For the case of one-body dissipation and friction in nuclear dynamics, Swiatecki and coworkers [2, 3, 4, 5] have developed this picture: particles which move in a shape-deformed container are reflected from the (moving) walls, and due to parts of it having positive curvature (for higher multipole moments) the particles very quickly loose their coherence, thus inducing pseudo-random motion, i.e. heat, into the system. At the same time the shape oscillation is very much slowed down.

Blocki *et al.* [4, 5] consider a purely classical gas of particles contained in a deformed billiard. The only similarity with a Fermi gas comes from the fact that initially the particles' momenta are distributed within a Fermi sphere. The walls of the container undergo periodic shape oscillations with a frequency much smaller than a typical single particle frequency. In the interior of the container the particles move on linear trajectories. They study the particle kinetic energy increase as a function of time and find that for ellipsoidal shape deformations ($\ell = 2$) the particles act as a classical Knudsen gas [6], i.e. the total kinetic energy increase over an entire shape oscillation period is 0. However, for $\ell \geq 3$ the kinetic energy in the single particle

motion is not completely 'given back', but rather steadily increases in time. This is explained by the fact that in an $\ell = 2$ potential the motion of the particles remains non-chaotic and therefore coherent, whereas in the $\ell \geq 3$ the scattering of the segments of the wall with positive curvature leads to chaotic motion similar to the one observed in a Sinai [7, 8] billiard and thus a destruction of coherence. The fact that deformed nuclear potentials may exhibit chaotic motion was recognized early by Arvieu and co-workers [9]

This scenario is very similar to the so-called Fermi acceleration, proposed to explain the occurrence of very high energy cosmic radiation [10, 11].

In this paper we present an attempt to include selfconsistency into the problem of motion in multipole-deformed nuclear potentials. We have chosen a selfconsistent, but schematic, model of separable forces. We chose an interaction of the Bohr-Mottelson type [12] with a static r^2 potential and multipole-multipole interactions as studied, for example, by Stringari *et al.* [13, 14, 15, 16]. In the small amplitude limit, this model has recently been investigated in the semiclassical limit [16]; as is known, the low-lying quadrupole and octupole frequencies come out to be in reasonable agreement with experimental data. Our single-particle Hamiltonian is then

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_0 + V^{(\ell)}(\boldsymbol{r}, t) \\ &= \frac{p^2}{2\,m} + V_0 + V^{(\ell)}(\boldsymbol{r}, t) \end{aligned} \tag{1}$$

$V^{(\ell)}(\boldsymbol{r},t)$ is the potential associated with the (separable) multipole-multipole force [13, 14, 16]

$$V^{(\ell)}(\boldsymbol{r},t) = \lambda_\ell \, q_\ell(\boldsymbol{r}) \, Q_\ell(t) \ , \tag{2}$$

and V_0 is the static external potential. We take here $V_0 = \frac{1}{2}\, m\, \omega_0^2 \, r^2$, resulting in the Bohr-Mottelson Hamiltonian [12]. However, we have also investigated non-harmonic static potentials (r^6) and obtained similar results.

The coupling constants λ_ℓ can be calculated using a self-consistent normalization condition [12, 16].

$$\lambda_{2,s} = -\frac{3\, m\, \omega_0^2}{A\langle r^2\rangle} \ , \tag{3}$$

$$\lambda_{3,s} = -\frac{15\, m\, \omega_0^2}{A\langle r^4\rangle} \ , \tag{4}$$

where A is the mass number of the nucleus under consideration. $q_\ell(\boldsymbol{r})$ is given by

$$q_2(\boldsymbol{r}) = r_y \, r_z \tag{5}$$

$$q_3(\boldsymbol{r}) = r_x \, r_y \, r_z \tag{6}$$

and the multipole moments $Q_\ell(t)$ are

$$Q_\ell(t) = \int \frac{d^3r \, d^3p}{(2\pi)^3} q_\ell(\boldsymbol{r}) f(\boldsymbol{r},\boldsymbol{p},t) \ . \tag{7}$$

$f(\boldsymbol{r},\boldsymbol{p},t)$ is the one-body phase space distribution function of nucleons, the Wigner transform of the one-body density.

We treat this problem in semi-classical approximation by a Wigner transformation of the von Neumann equation of motion for the density matrix, $i\,\partial_t \rho = [\mathcal{H}, \rho]$, to obtain a Vlasov equation, $\partial_t f = \{\mathcal{H}, f\}$. We then solve the Vlasov equation in the test particle method [17, 18] using a fourth-order Runge-Kutta algorithm with typical time step sizes of 1 fm/c. Our numerical simulation is fully selfconsistent and conserves total energy to better than 0.1 %.

In order to generate selfconsistent initial deformations in coordinate and momentum space, we start with a spherically symmetric configuration generated in local Thomas-Fermi approximation without the deformation potential $V^{(\ell)}$.

We now apply a time-dependent external potential of the form

$$V_0^{(\kappa)}(\ell, \boldsymbol{r}, t) = \kappa_0 \,\sin(\omega_D\, t)\, s(t)\, q_\ell(\boldsymbol{r}) \tag{8}$$

where ω_D is the driving frequency, and where $s(t)$ is a differentiable spline interpolation function on the time interval $[0,\tau]$ with vanishing first derivatives at both ends, which is monotonically increasing from 0 to 1. This procedure results in a giant oscillation of the nucleus at $t = \tau$, provided that τ is chosen $\tau \gg \omega_D^{-1}$. The deformation is dependent on the value of the coupling κ_0 chosen.

We now use the initial conditions generated in this way (with $\tau = 1500$

fm/c) to study the time evolution under the action of our Hamiltonian as defined in Eq. (1). The upper panel (a) of Figure 1 contains the results of our calculations. One can clearly observe a regular undamped oscillation of the quadrupole moment in coordinate space as a function of time. One can observe that the period of oscillation has been stretched from the 0-coupling value of 88.4 fm/c to 128 fm/c. This is consistent with the analytic calculations for infinitesimal deformations [19, 12, 16], which yield

$$\begin{aligned} Q_2(t) &= Q_2(t_0)\,\exp(i\,\omega_{2^+}\,t) \\ \omega_{2^+} &= \sqrt{4\omega_0^2 + \lambda_2 \frac{2\,A\,\langle r^2\rangle}{3m}} \\ &= \sqrt{2}\,\omega_0 \end{aligned} \tag{9}$$

for the giant quadrupole frequency, and consequently $T = 125$ fm/c for $\omega_0 = 0.0355$ c/fm used in this example.

In the lower panel (b) of Figure 1 we show our calculations for the octupole case. Again we see harmonic oscillations (The small variations in amplitude are in both cases due to beating between the initial driving frequency and the oscillation frequency of the self-consistent calculation.). The observed oscillation period is $T_3 \approx 66$ fm/c, in good agreement with the analytical result of [16]

$$\omega_{3^-} = \sqrt{7}\,\omega_0 \tag{10}$$

for the giant octupole oscillation frequency, which results in $T_3 \approx 66.8$ fm/c

for our value of ω_0.

The most important observation is here, however, that there is no damping of the collective motion apparent in our calculation, thus indicating that no chaoticity is present in the collective multipole coordinates. We have also used slightly different initial conditions (by using a different number of test particles in the simulation) and obtained only slightly different results. This indicates that there is no sensitive dependence on the initial conditions present here as would be the case for chaotic motion. As an additional test, we performed a Fourier transform of the time signal and found one peak at the dominant frequency and no ω^{-1} noise.

To study the single particle dynamics we employ the method of Poincaré surface of sections. In Poincaré sections, we stroboscopically record the phase space coordinates of particles in time. This 'stroboscope' is triggered whenever the particles cross a certain plane in phase space. This technique is also applicable to our problem, where we solve the Vlasov equation utilizing the test particle method. Here we choose the plane $r_x = 0$ as the trigger condition.

In Figure 2 (a) we show the Poincaré section for one test particle used to generate the octupole motion of Fig. 1 (b). One can clearly see that this Poincaré section will become area-filling in certain regions of phase space, and in the limit of very long time scales. (For the present calculations we ran

10^5 time steps of 1 fm/c each, leading to approximately 550 crossings of the $r_x = 0$ plane from below). In order to enable the reader to better compare the structures, we superimpose in Figure 2 (b) the Poincaré sections of ten test particles with different initial conditions (but parts of the same simulation). We obtain similar results for the collective quadrupole motion.

This chaoticity is not a result of the weak destruction of the integrability of the corresponding static multipole potential, which can be shown to exist for the static octupole (but not for the static quadrupole) potential. Instead we attribute the chaoticity in the single particle motion to the exchange of energy between the motion of the individual test particles and the collective motion of the multipole coordinate. This exchange of energy is possible, because the individual test particles oscillate with frequencies, which do not have a rational ratio with the frequency of the collective coordinate. This results in the particle reaching meta-stable or unstable points in phase space during the course of its time evolution. At these points small changes in the initial conditions will have a large effect on the subsequent dynamics. An example for this would be the decision if the particle will temporarily oscillate in or out of phase with the collective coordinate. Consequently, these points provide large positive contributions to the Kolmogorov entropy, and chaotic single particle dynamics results.

In turn one also expects each single test particle to have a randomly

fluctuating effect on the energy contained in the motion of the collective coordinate. However, since there are quite many test particles, these chaotic random fluctuations are averaged out leaving only a smooth sinusoidal oscillation of the collective coordinate. This is qualitatively new in our investigation: the generation of regular dynamics for the collective variable, the multipole moment of the collective oscillation, from the ensemble of single particles with chaotic trajectories. This is an example of how ordered macroscopic motion can result form underlying chaotic microscopic dynamics. (To obtain this result, it was crucial to employ a self-consistent treatment of the dynamics entailing conservation of total energy.)

Our finding are not restricted to the static harmonic potential, which we used as an example here, but they hold for a general class of central potentials, V_0. We have also performed calculations for a $V_0 \propto r^6$ potential with very similar results to the ones presented here [20].

And, finally, one may speculate that this interplay between chaoticity in individual single particle degrees of freedom and regularity in certain collective coordinates may also play a role in the time evolution of other physical systems. Examples that come to mind as likely candidates are plamas in a tokamak, the human brain wave activity, the weather. Chaos on a microscopic level need not necessarily lead to a catastrophic breakdown of the system on the macroscopic scale.

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of the destruction of integrability in the static octupole potential will be published elsewhere.

Figure Captions

Figure 1

Time evolution of quadrupole (a) and octupole (b) deformations of an A=200 nucleus in coordinate space under propagation with the Hamiltonian of Eq. (1) with a static harmonic oscillator potential and self-consistent coupling strength $\lambda_2 = \lambda_{2,s} = 8.9 \cdot 10^{-4}$ MeV fm^{-4} (a), and $\lambda_3 = \lambda_{3,s} = 1.3 \cdot 10^{-4}$ MeV fm^{-6} (b).

Figure 2

Poincaré section of a single test particle (a) and an ensemble of 10 test particles (b) from the calculation leading to the time evolution of the octupole moment shown in Fig. 1 (b).

This figure "fig1-1.png" is available in "png" format from:

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